

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science; Bachelor of Science in Applied Mathematics and Statistics			
QUALIFICATION CODE:	07BOSC; 07BSAM	LEVEL:	5
COURSE CODE:	CLS502S	COURSE CODE:	CALCULUS 1
SESSION:	JANUARY 2023	PAPER:	THEORY
DURATION:	3 HOURS	MARKS:	100

SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION QUESTION PAPER		
EXAMINER:	DR. DSI IIYAMBO	
MODERATOR:	DR. N CHERE	

INSTRUCTIONS

- 1. Attempt all the questions in the booklet provided.
- 2. Show clearly all the steps used in the calculations.
- 3. All written work must be done in black or blue inked, and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Question 1.

Consider the functions $f(x) = 4x^2 + 9$, $g(x) = \sqrt{1-x^2}$ and $h(x) = 4x^2 - 3$; $x \ge 0$.

- a) Find the sum of the smallest and the largest numbers in the domain of $\frac{g}{f}$. [9]
- b) Determine whether g is even, odd or neither. [4]
- c) Determine whether h^{-1} exists. If it does, find it. [10]

Question 2.

a) Find the following limits, if they exist.

(i)
$$\lim_{x \to -3} \frac{4x + 12}{x^3 + 3x^2 - 4x - 12}$$
. [5]

(ii)
$$\lim_{x \to 0^+} (e^x + x)^{\frac{1}{x}}$$
 [8]

b) Use the $\varepsilon - \delta$ method to show that $\lim_{x \to 2} (10x - 6) = 14$. [7]

Question 3.

- a) Use the definition (first principle) to find the derivative of $f(x) = \frac{1}{\pi^2} \frac{x-1}{x+\sqrt{2}}$. [10]
- b) Differentiate the function $f(x) = (\ln 3)^{\sec x} + \tan^{-1}(\ln 4x)$. [6]
- c) If the equation $x^2y + \sin y = 2\pi$ determines a differentiable function f such that y = f(x), find the equation of the tangent line to the graph of the given equation at the point $P(1, 2\pi)$.

[8]

Question 4.

Let f(x) = |2x - 10| + 2.

- a) Show that f is continuous at x = 5. [7]
- b) Show that f is not differentiable at x = 5.

Question 5.

Let $f(x) = \frac{x^4}{4} - 2x^2 + 4$ and $g(x) = 2x^4 - 8x^3 + 316x - 172$.

- a) Find the intervals on which f is increasing and on which it is decreasing. [9]
- b) Find the intervals on which the graph of y = g(x) is concave upwards and on which it is concave downwards. [9]

END OF EXAMINATION QUESTION PAPER